

# AP CALCULUS BC SUMMER ASSIGNMENT

Edmodo.com: AP Calculus BC 2017-2018 Group Code: **kdw69v**

Attached is an assignment for students entering AP Calculus BC in the fall. Next year we will focus more on concepts and thinking outside of the box. We will not have time to review basic algebraic concepts that you should already know. Therefore it is imperative that you have mastered the basic skills needed for success in this course. Many students struggle with the Algebra component of Calculus and neglect the concepts and theorems. It is a shame when a student understands the calculus but can't simplify, thus getting the problem incorrect. Thus the purpose of this worksheet is to help you be better prepared for next year.

Also included are basic concepts you must know. You may want to use your notes from Pre-Calculus as well. Although the assignment says BC, the introductory concepts are the same.

**YOU MUST COMPLETE THE FOLLOWING PROBLEMS ON A SEPARATE SHEET OF PAPER. WORK SHOULD BE NEAT AND EASILY FOLLOWED.**

This will be checked on the first day of school and will be collected on the second. I will be checking for completeness and correctness. There should be evidence that you have checked your work. A key will be provided (posted on Edmodo.com) by August 1st. Put an **X** by the problems that you got wrong but corrected. Put a **?** by the problems you got wrong but had difficulty correcting. There will be a **TEST** over this material on the **second day of school (Friday - August 18<sup>th</sup>)!** There will be a Q & A session in the afternoon of Wednesday (August 16<sup>th</sup>) and in class Thursday (August 17<sup>th</sup>). Details will be sent out on Edmodo.com.

You should wait till mid-summer to begin this packet. This will give you a better indication of what you know and what you've forgotten. If you have any questions, contact me at [apugeda@psd202.org](mailto:apugeda@psd202.org) or on Edmodo.com (Edmodo.com is a little better because I will receive notifications on my phone and I rarely check my work email during the summer). Keep in mind that this packet is not to "torture" you or "punish" you for taking AP Calculus BC. This will better prepare you and allow us more time to review before the exam.

**Enjoy your summer,**

**P**

## DO I BELONG IN AP CALCULUS BC??

### HERE ARE MR P'S RECOMMENDATIONS

1. Took CalcAB and was recommended by Mr. P – Definitely.
2. Took CalcAB but was not recommended by Mr. P – Will be ok 1<sup>st</sup> semester...2<sup>nd</sup> semester will be rough.
3. 95%-100% in Honors Pre-Calculus – Yes.
4. 90%-94% in Honors Pre-Calculus – Should be ok. But its gonna be a rough ride. (Might be more comfortable in AB)
5. 89% or lower in Honors Pre-Calculus – Not recommended.

## AP CALCULUS BC SUMMER ASSIGNMENT

Complete the following. Show and attach all work in a clear manner. Do NOT do any work on this sheet – all work must be separate. Have this assignment completed and ready to turn in on the first day of school. No calculators are allowed except where noted.

#1-6. Are the following statements true? If not, explain IN COMPLETE SENTENCES why not?

1.  $\frac{2k}{2x+h} = \frac{k}{x+h}$

2.  $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$

3.  $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

4.  $3\frac{a}{b} = \frac{3a}{3b}$

5.  $3\frac{a}{b} = \frac{3a}{b}$

6.  $3\frac{a+b}{c} = \frac{3a+b}{c}$

#7-16. Factor each of the following completely.

7.  $a^2 - b^2$

8.  $a^3 - b^3$

9.  $8x^3 + y^3$

10.  $4x^2 - 21x - 18$

11.  $2x^2 + x - 3$

12.  $3x^2 + 6x^3 - 9x$

13.  $(x+1)^3(4x-9) - (16x+9)(x+1)^2$

14.  $(x-1)^3(2x-3) - (2x+12)(x-1)^2$

15.  $(2x-1)^2(x-3) + (x+1)(2x-1)^3$

16. Factor  $(x-a)$  in such a way that  $\sqrt{x} - \sqrt{a}$  is a factor.

#17-24. Simplify.

17.  $\frac{x/2}{x/4}$

18.  $h \div \frac{(x+h)}{h}$

19.  $\frac{\sqrt{x-2} + \frac{5}{\sqrt{x-2}}}{x-2}$

20.  $\frac{x(5x+1) - 3(x^2+1)}{(x-1)^2}$

21.  $\frac{(x+1)^3(4x-9) - (16x+9)(x+1)^2}{(x-6)(x+1)^3}$

22.  $\frac{3x(x+1) - 2(2x+1)}{(x-1)^2}$

23.  $\frac{2x(x+1)^2 - 3(x+1)^3}{8x^3 + 30x^2 + 18x}$

24.  $\frac{(x-1)^3(2x-3) - (4x-1)(x-1)^2}{(x-1)^2(2x-1)}$

**#25-28. Solve the equation.**

25.  $4x^2 - 21x - 18 = 0$

26.  $x^3 + 3x^2 - 5x - 15 = 0$

27.  $x^4 - 9x^2 + 8 = 0$

28.  $4 - 3^x = 0$

29. Write as a single fraction with denominator in factored form:  $\frac{7x^2+5x}{x^2+1} - \frac{5x}{x^2-6} = 0$

30. Graph the equation  $y = x^3 - x$  and answer the following questions.

a) Is the point (3, 2) on the graph?

b) Is the point (2, 6) on the graph?

c) Is the function even, odd, or neither?

d) What is the y-intercept?

e) Find the x-intercepts.

31. Find ALL intercepts of the graph of  $y = \frac{x-1}{x+3}$

**#32-35. Show work to determine if the relation is even, odd or neither.**

32.  $f(x) = 2x^2 - 7$

33.  $f(x) = -4x^3 - 2x$

34.  $f(x) = 4x^2 - 4x + 4$

35.  $f(x) = \frac{x^2}{x^2-4}$

36. Find the equation of the straight line that passes through the point (2, 4) and is parallel to the line  $2x + 3y - 8 = 0$

37. Find the equation of the line that is perpendicular to the line  $2x + 3y - 8 = 0$  at the point (1, 2).

38. The line with the slope 5 that passes through the point  $(-1, 3)$  intersects the  $x$ -axis at a point. What are the coordinates of this point?

39. What are the coordinates of the point at which the line passing through the points  $(1, -3)$  and  $(-2, 4)$  intersects the  $y$ -axis?

40. Given:  $f(x) = |x - 3| - 5$  Find  $f(1) - f(5)$ .

41. Find all points of intersection of the graphs of  $x^2 + 3x - y = 3$  and  $x + y = 2$

42. If the point  $(-1, 1)$  lies on the graph of the equation  $kx^2 - xy + y^2 = 5$ , find the value of  $k$ .

43. Write the equation of a graph that is a function.

44. Write the equation of a graph that is NOT a function.

**#45-48. Find the domain for each of the following functions.**

45.  $h(x) = \frac{1}{4x^2 - 21x - 18}$

46.  $k(x) = \sqrt{x^2 - 5x - 14}$

47.  $p(x) = \frac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

48.  $y = \ln(2x - 12)$

49. For the function  $y = 5 - \sqrt{9 - x^2}$ , a) find the domain, b) find the range, and c) determine whether the function is odd, even, or neither.

50. Let  $f(x) = \begin{cases} -0.5x & x < -2 \\ \sqrt{x+2} & x \geq -2 \end{cases}$  a) draw the graph of  $f(x)$ . b) find the domain, c) find the range.

51. Find  $f(x + \Delta x)$  for  $f(x) = x^2 - 2x - 3$

**#52-53. Sketch the graph of each function.**

$$52. f(x) = \begin{cases} 1, & x \leq 0 \\ -1, & x > 0 \end{cases}$$

$$53. f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x + 3 & (2, \infty) \end{cases}$$

54. State the domain, range and intercepts of the function  $y = 2^{-x} - 1$ .

**#55-57. Given  $f(x) = x - 3$  and  $g(x) = \sqrt{x}$  complete the following**

$$55. f(g(x)) =$$

$$56. g(f(x)) =$$

$$57. f(f(x)) =$$

**#58-60. Given  $f(x) = \frac{1}{x-5}$  and  $g(x) = x^2 - 5$  complete the following**

$$58. f(g(7)) =$$

$$59. g(f(v)) =$$

$$60. g(g(x)) =$$

**#61-64. Let  $f(x) = 2x - 2$ . Complete the following:**

61. Sketch the graph of  $f(x)$ .

62. Determine whether  $f$  has an inverse function. How do you know?

63. Sketch the graph of  $f^{-1}(x)$ .

64. Give the equation for  $f^{-1}(x)$ .

**#65-66. Simplify using only positive exponents. Do not rationalize the denominator.**

$$65. \frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$$

$$66. \left( \frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}} \right)^{-\frac{1}{2}}$$

**#67-72. If  $f(x) = x^2 - 1$ , describe in words what the following would do to the graph of  $f(x)$ .**

$$67. f(x) - 4$$

$$68. f(x - 4)$$

$$69. -f(x + 2)$$

$$70. 5f(x) + 3$$

$$71. f(2x)$$

$$72. |f(x)|$$

73. (**calculator**) The dollar value of a product in 1998 is \$78. The value of the product is expected to decrease \$5.75 per year for the next 5 years. Write a linear equation that gives the dollar value  $V$  of the product in terms of the year  $t$ . (Let  $t = 8$  represent 1998.)

74. (**calculator**) A business had annual retail sales of \$124,000 in 1993 and \$211,000 in 1996. Assuming that the annual increase in sales follows a linear pattern, predict the retail for 2007.

75. (**calculator**) In order for a company to realize a profit in the manufacture and sale of a certain item, the revenue,  $R$ , for selling  $x$  items must be greater than the cost,  $C$ , of producing  $x$  items. If  $R = 69.99x$  and  $C = 59x + 850$ , for what values of  $x$  will this product return a profit?

76. (**calculator**) Suppose that in any given year, the population of a certain endangered species is reduced by 25%. If the population is now 7500, in how many years will the population be 4000?

77. A piece of wire 5 inches long is to be cut into two pieces. One piece is  $x$  inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of  $x$ .

78. Let  $f(x) = \sqrt[3]{x+2}$  and  $g(x) = x^3 - 2$ . Which of the following are true?

I.  $g(x) = f^{-1}(x)$  for all real values of  $x$ .

II.  $(f \circ g)(x) = 1$  for all real values of  $x$ .

III. The function  $f$  is one to one.

79. Let  $f(x) = \sqrt{3-x}$ . Find an expression for  $f^{-1}(x)$ . (Be sure to state any necessary domain restrictions.)

80. Let  $y = 3 \sin(2x - \pi) + 2$ . Determine the **period**, **domain**, and **range** of the function.

**#81-88. Evaluate. (Answers must be exact and answers for #90 and 91 must be in radians).**

81.  $\cos(0)$

82.  $\sin(0)$

83.  $\tan\left(\frac{\pi}{2}\right)$

84.  $\cos\left(\frac{\pi}{4}\right)$

85.  $\sin\left(\frac{\pi}{2}\right)$

86.  $\sin(\pi)$

87.  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

88.  $\arctan(1)$

**#89-91. Find the solution of the equations for  $0 \leq x < 2\pi$**

89.  $2 \sin^2(\theta) = 1 - \sin(\theta)$

90.  $2 \tan(\theta) - \sec^2(\theta) = 0$

91.  $\sin(2\theta) + \sin(\theta) = 0$

92. Which of the following expressions are identical?

a)  $\cos^2 x$       b)  $(\cos x)^2$       c)  $\cos x^2$

93. Which of the following expressions are identical?

a)  $(\sin x)^{-1}$       b)  $\arcsin(x)$       c)  $\sin x^{-1}$       d)  $\frac{1}{\sin x}$

**#94-100. Solve for  $x$ .**

94.  $\ln e^3 = x$

95.  $\ln e^x = 4$

96.  $\ln x + \ln x = 0$

97.  $e^{\ln 5} = x$

98.  $\ln 1 - \ln e = x$

99.  $\ln 6 + \ln x - \ln 2 = 3$

100.  $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

**#101-111. Evaluate the limit.**

101.  $\lim_{x \rightarrow 2} (3x^2 + 5)$

102.  $\lim_{x \rightarrow -1} \frac{x^4 + x^3}{x + 1}$

103.  $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$

104.  $\lim_{x \rightarrow -2} \frac{x - 4}{x^2 - 2x - 8}$

105.  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$

106.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

107.  $\lim_{x \rightarrow 2} \sec\left(\frac{\pi x}{3}\right)$

108.  $\lim_{x \rightarrow 3^+} \sqrt{2x - 5}$

109.  $\lim_{x \rightarrow 2^-} \frac{1}{x - 2}$

110.  $\lim_{x \rightarrow 1} f(x)$  if  $f(x) = \begin{cases} 3 - x, & x \neq 1 \\ 1, & x = 1 \end{cases}$

111.  $\lim_{x \rightarrow 5} \frac{x - 5}{|x - 5|}$



#112-114. For each of the following, determine: a)  $\lim_{x \rightarrow 1^-} f(x)$  b)  $\lim_{x \rightarrow 1^+} f(x)$  c)  $\lim_{x \rightarrow 1} f(x)$

$$112. f(x) = \begin{cases} x^2 - 1, & x < 1 \\ 4 - x, & x \geq 1 \end{cases} \quad 113. f(x) = \begin{cases} 3x - 1, & x \leq 1 \\ 3 - x, & x > 1 \end{cases} \quad 114. f(x) = \begin{cases} -x^2 & x < 1 \\ 2 & x = 1 \\ x - 2 & x > 1 \end{cases}$$

#115-125. Answer the following.

115. At which values of  $x$  is  $f(x) = \frac{(x+1)(x-3)}{x-2}$  discontinuous?

116. Determine the value of " $c$ " so that  $f(x)$  is continuous on the entire real line.  $f(x) = \begin{cases} x - 2, & x < 5 \\ cx - 3, & x \geq 5 \end{cases}$

117. Let  $f(x) = \frac{1}{x+1}$  and  $g(x) = x^2 - 5$ . Find all values of  $x$  for which  $f(g(x))$  is discontinuous.

118. Find all vertical asymptotes of  $g(x) = \frac{x-3}{x+2}$ .

119. Find all vertical asymptotes of  $g(x) = \frac{x+1}{x^2-1}$ .

120. Use the "definition of a derivative" to find  $f'(x)$  if  $f(x) = 2x^2 + 3x - 5$

121. Use the differential formulas ("shortcuts") to find the derivative of  $y = 5x^3 - 7x^2 + 2x - 9$

122. Use the differential formulas to find the derivative of  $y = \frac{1}{\sqrt{x}}$ . (No negative exponents in final answer.)

123. Use the differential formulas to find the derivative of  $f(x) = \frac{3x+5}{2x-1}$ .

124. Use the differential formulas to find the derivative of  $y = \frac{\sqrt{x}}{x^2+1}$

125. Find the equation of the line tangent to  $f(x) = x^2 + 4x + 3$  when  $x = 1$ .