Quadrilaterals and Their Properties
A 4-gon Hypothesis
Lesson 15-1 Kites and Triangle Midsegments

Learning Targets:
- Develop properties of kites.
- Prove the Triangle Midsegment Theorem.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Shared Reading, Create Representations, Think-Pair-Share, Interactive Word Wall, Group Presentations

Mr. Cortez, the owner of a tile store, wants to create a database of all the tiles he sells in his store. All of his tiles are quadrilaterals, but he needs to learn the properties of different quadrilaterals so he can correctly classify the tiles in his database.

Mr. Cortez begins by exploring convex quadrilaterals. The term quadrilateral can be abbreviated “quad.”

1. Given quad GEOM.
   a. List all pairs of opposite sides.
      \[ \overline{GM} \neq \overline{EO}, \overline{GE} \neq \overline{OM} \]
   b. List all pairs of consecutive sides.
      \[ \overline{GM} \neq \overline{MO}, \overline{MO} \neq \overline{EO}, \overline{EO} \neq \overline{GE}, \overline{GE} \neq \overline{GM} \]
   c. List all pairs of opposite angles.
      \[ \angle G \neq \angle O, \angle E \neq \angle M \]
   d. List all pairs of consecutive angles.
      \[ \angle G \neq \angle M, \angle M \neq \angle O, \angle O \neq \angle E, \angle E \neq \angle G \]
   e. Draw the diagonals, and list them.
      \[ \overline{GO} \neq \overline{EM} \]
A *kite* is a quadrilateral with exactly two distinct pairs of congruent consecutive sides.

2. Given quad $KITE$ with $\overline{KI} \cong \overline{KE}$ and $\overline{IT} \cong \overline{ET}$.
   a. One of the diagonals divides the kite into two congruent triangles. Draw that diagonal and list the two congruent triangles. Explain how you know the triangles are congruent.

   $\triangle KIT \cong \triangle KET$ by SSS

   b. Draw the other diagonal. Explain how you know the diagonals are perpendicular.

   c. Complete the following list of properties of a kite. Think about the angles of a kite as well as the segments.

   1. Exactly two pairs of consecutive sides are congruent.
   2. One diagonal divides a kite into two congruent triangles.
   3. The diagonals of a kite are perpendicular.
   4. *diagonal* bisects *kite* into 2 $\triangle$s.
   5. Pair opp. $\angle$s $\cong$.
   6. *diagonal* bisects pair opp. $\angle$s.

3. **Critique the reasoning of others.** Mr. Cortez says that the diagonals of a kite bisect each other. Is Mr. Cortez correct? Support your answer with a valid argument.

   *Only 1 diagonal is bisected*

**Check Your Understanding**

4. Why is a square not considered a kite?

5. Suppose $\overline{AC}$ and $\overline{BD}$ are the diagonals of a kite. What is a formula for the area of the kite in terms of the diagonals?

   Area = $\frac{1}{2}$ diagonal $\cdot$ diagonal
Lesson 15-1
Kites and Triangle Midsegments

The segment whose endpoints are the midpoints of two sides of a triangle is called a **midsegment**.

**Triangle Midsegment Theorem** The midsegment of a triangle is parallel to the third side, and its length is one-half the length of the third side.

6. Use the figure and coordinates below to complete the coordinate proof for the Triangle Midsegment Theorem.

![Figure with coordinates](image)

a. Complete the hypothesis and conclusion for the Triangle Midsegment Theorem.

**Hypothesis:**
- \( M \) is the midpoint of \( AB \).
- \( N \) is the midpoint of \( BC \).

**Conclusion:**
- \( MN \parallel AC \)
- \( MN = \frac{1}{2} AC \)

b. Find the coordinates of midpoints \( M \) and \( N \) in terms of \( a, b, c, h, k, \) and \( l \).

\[
m, M = \left( \frac{a+b}{2}, \frac{h+k}{2} \right) \quad m, N = \left( \frac{b+c}{2}, \frac{k+l}{2} \right)
\]

c. Find the slope of \( AC \) and \( MN \).

\[
\frac{m, AC}{m, MN} = \frac{\frac{e-h}{c-a}}{\frac{k+l}{a-b}} = \frac{\frac{k+l}{a-b} - \frac{h+k}{c-a}}{\frac{e-h}{c-a}}
\]

d. Simplify your response to part c and explain how your answers to part c show \( MN \parallel AC \).

**slopes are same**

e. Find \( AC \) and \( MN \).

\[
d, AC = \sqrt{(c-a)^2 + (e-h)^2}
\]

\[
d, MN = \sqrt{\left( \frac{b+c}{2} - \frac{a+b}{2} \right)^2 + \left( \frac{k+l}{2} - \frac{h+k}{2} \right)^2}
\]

f. Simplify your response to part e and explain how your answers to part e show that \( MN = \frac{1}{2} AC \).