LEsson 9-1

1. Examine the data in the table. What type of function could be used to model the data? Explain your reasoning.

<table>
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</thead>
<tbody>
<tr>
<td>Number of Enrolled Students (in thousands)</td>
<td>10</td>
<td>10.5</td>
<td>12</td>
<td>15.6</td>
<td>20.1</td>
<td>21.2</td>
<td>22.7</td>
<td>25</td>
<td>30.5</td>
<td>32.8</td>
<td></td>
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</tbody>
</table>

2. **Model with mathematics.** Use the regression capabilities of your graphing calculator to find a model that best represents the data in Item 1.

3. **Attend to precision.** Graph the equation you found in Item 2. List the important features of the graph. Approximate any values to three decimal places.

4. Use the regression model to predict about how many students will be attending University XYZ in 2020.
   
   - **A.** 34,000
   - **B.** 36,000
   - **C.** 38,000
   - **D.** 2,899,000

5. Data are collected on puppies and their growth (by weight) over a 6-month period. What type of function could be used to model the data for the weight as a function of the number of months since the birth of a puppy? Explain your reasoning.

LESson 9-2

6. Which of the following functions is a fifth degree polynomial?

   - **A.** \( f(x) = x(x - 3)^2(x + 2) \)
   - **B.** \( f(x) = x^2 + 2x(x + 5)(x - 7) \)
   - **C.** \( f(x) = (x - 1)(x + 9)^3(x - 2) \)
   - **D.** \( f(x) = x^5(x - 11) \)
7. **Make use of structure.** List the important features of the graph below. Approximate any values to three decimal places.

![Graph of a function with coordinates]

8. Without using a calculator, determine the end behavior and x- and y-intercepts of the function $f(x) = (2x - 1)(x + 1)(x + 3)$.

9. Without using a calculator, find the end behavior, maximum possible zeros, and maximum possible turning points of the function $f(x) = x^6 - 2x^3 + 1$.

10. **Use appropriate tools strategically.** Use a graphing calculator to find the zeros, turning points, y-intercepts, and end behavior.

\[ y = 2x^5 + 5.7x^4 - 8x^3 - 15x^2 + 28 \]

**LESSON 10-1**

**Make use of structure.** For Items 11 and 12, determine the y-intercept and the end behavior of each function.

11. $f(x) = 7x^7 - 8x^6 + 3x^4 + 2x^2 - 1$

12. $h(x) = 4(x - 1)^3(x - 3)(x + 2)$

13. What are the zeros of the function $f(x) = (x + 2)^4(x - 3)^5$?
   - A. $x = 2, -3$
   - B. $x = -2, 3$
   - C. $x = 4, 5$
   - D. $x = -4, -5$

14. **Attend to precision.** Factor and find the zeros of the function $g(m) = m^4 - 2m^3 + 8m - 16$.

15. Graph $f(x) = (x - 1)(x + 5)^2$. 

![Graph of a function with coordinates]
Lesson 10-2

16. Which of the following are possible zeros of \( f(x) = 3x^4 + x^2 - 8 \)?

A. \( \pm \frac{8}{3} \)

B. \( \pm \frac{3}{2} \)

C. \( \pm \frac{1}{2} \)

D. \( \pm \frac{16}{3} \)

17. Use the Rational Root Theorem to find the possible real zeros and the Factor Theorem to find the zeros of the function.

\[ u(t) = 3t^3 - 5t^2 + 6t + 8 \]

Make use of structure. For Items 18 and 19, use the Rational Root Theorem and synthetic division to find the real zeros.

18. \( f(x) = 35x^3 - 114x^2 + 25x + 6 \)

19. \( g(x) = 4x^4 - 65x^3 + 16 \)

20. Use appropriate tools strategically. Use the Rational Root Theorem and a graphing calculator to find the rational root(s) of the polynomial.

\( f(x) = x^4 - 5x + 4 \)

Lesson 10-3

21. Which function has the greatest number of sign changes?

A. \( f(x) = x^7 - 10x^6 + 40x^5 - 96x^4 + 176x^3 - 224x^2 - 128x - 1 \)

B. \( g(x) = x^7 - 10x^6 - 40x^5 - 96x^4 + 176x^3 - 224x^2 - 128x - 1 \)

C. \( h(x) = x^7 + 10x^6 - 40x^5 - 96x^4 - 176x^3 + 224x^2 - 128x + 1 \)

D. \( j(x) = x^7 + 10x^6 + 40x^5 + 96x^4 + 176x^3 + 224x^2 + 128x - 1 \)

Make use of structure. For Items 22 and 23, determine the number of positive and negative real zeros.

22. \( f(x) = -120x^3 - 146x^4 - x^3 + 27x^2 + x - 1 \)

23. \( f(x) = -150x^3 + 153x + 90 \)

Attend to precision. For Items 24 and 25, sketch a graph of the polynomial function.

24. \( f(x) = x^4 - 3x^3 + 3x - 1 \)
25. \( f(x) = 2x^4 - 3x^3 - 2x^2 + 3x + 1 \)

b. Sketch and label a graph of the volume function.

LES Son 11-1
Orange Cell Phone Company is creating a package for their newest phone. The box package is made from cardboard pieces that are 14 inches long by 5 inches wide. The boxes are made by cutting a section of size \( x \) by \( x \) out of each corner of the cardboard.

26. Model with mathematics. Write the equation that represents the volume of the box.

27. a. Determine the possible domain and range for the construction of these boxes.

28. Use a graphing calculator to find the maximum volume of a box that the Orange Cell Phone Company can make. What are the dimensions of the box with a maximum volume, and what is its volume?

29. Which polynomial has degree 6 and zeros \( x = -6, 1, 0, \frac{1}{2}, 1, \frac{3}{2} \)?

A. \( f(x) = (x - 1)^2(x - 3)(x + 1)(x + 6) \)
B. \( f(x) = x \left( x + \frac{1}{2} \right)(x + 1) \left( x + \frac{3}{2} \right)(x - 1)(x - 6) \)
C. \( f(x) = x(2x + 1)(x + 1)(2x + 3)(x - 1)(x - 6) \)
D. \( f(x) = x(2x - 1)(x - 1)(2x - 3)(x + 1)(x + 6) \)

30. Make use of structure. Find a polynomial with real coefficients of given degree with the given zeros.
degree: 4; zeros: \( x = -1, 0, 3, 10 \)
LESSON 11-2

31. Which of the following is a factor of the function \( f(x) = 2x^3 - 8 \)?
   A. \( x + 8 \)  
   B. \( x - 4 \)  
   C. \( x - 2 \)  
   D. \( 2x - 2 \)

Make use of structure. For Items 32 and 33, rewrite each polynomial function as a product of complex factors.

32. \( f(x) = x^2 + 144 \)

33. \( g(x) = 3x^4 - 4x - 7 \)

Attend to precision. For Items 34 and 35, find the zeros of each function.

34. \( h(x) = x^2 - 2x + 3 \)

35. \( r(x) = x^4 + 27x \)

LESSON 11-3

Model with mathematics. MetalBox Manufacturing also makes industrial air conditioning frames from an 8-foot-by-15-foot piece of metal. Square corners of length \( x \) are cut from each piece. The volume of the frame box must be at least 28 cubic feet.

36. Write an inequality for the volume that satisfies the constraint.

37. Use a graphing calculator to determine the interval over which the volume of the frame boxes made from the pieces of metal is larger than 28 cubic feet.

38. Use appropriate tools strategically. Using the piece of metal, what are the dimensions of the air conditioner with the largest possible volume?
   A. 1.7 feet by 4.6 feet by 5.8 feet  
   B. 2 feet by 8 feet by 15 feet  
   C. 2 feet by 4 feet by 3.5 feet  
   D. 1.7 feet by 7.5 feet by 8 feet

Solve each inequality and write the solution interval.

39. \( x^2 - 5x < 6 \)

40. \( x^4 \geq 324 \)

LESSON 12-1

Model with mathematics. Student A and Student B are studying for the same test at the same rate. Both students studied through the dinner hour. Right now, Student B has been studying twice as many hours after the dinner hour as Student A. When both students study 1 more hour, Student B will have studied one and a half times as many hours from the dinner hour as Student A.

Let \( h \) represent Student A’s number of hours studied since the dinner hour.

41. Which equation can be used to find \( h \), the current number of hours Student A has been studying?
   A. \( h = h + 1 \)  
   B. \( h + 1 = 1.5(h + 1) \)  
   C. \( 2h = 2h + 1 \)  
   D. \( 2h + 1 = 1.5(h + 1) \)
42. Find the number of hours Student A has studied right now since the dinner hour.

43. Let $S(x)$ represent the ratio of Student A’s number of hours studied since the dinner hour to Student B’s number of hours studied since the dinner hour, and let $x$ represent the number of hours from now, either past or future. Write $S$ as a function of $x$.

44. What appears to happen to the ratio of Student A’s number of hours since the dinner hour to Student B’s number of hours since the dinner hour as $x$ increases?

45. Reason abstractly. If the two students keep studying at the same rate forever, would Student A ever catch up to Student B? Explain.

LESSON 12-2

Model with mathematics. $O(x) = \frac{10 + x}{13 + x}$ represents the ratio of Amy’s age in years to Michael’s age in years, where $x$ represents the number of years from now, either past or future.

46. Sketch a graph of the function $O(x)$ for $-15 < x < 15$.

47. Find the equation of the vertical asymptote of $O(x)$.
   A. $x = 1$
   B. $x = -1$
   C. $x = 13$
   D. $x = -13$

48. Find the equation of the horizontal asymptote of $O(x)$.
   A. $y = 1$
   B. $y = -1$
   C. $y = 10$
   D. $y = -13$

49. Demonstrate why the value of $O(x)$ will never actually reach the value of the horizontal asymptote.

50. Use appropriate tools strategically. Use a graphing calculator to look at the behavior of the $O(x)$ to the left of the vertical asymptote. Explain why you are not looking at that part of the graph for the given scenario.
LESSON 13-1

51. Consider the graphs for \( f(x) = \frac{1}{x} \) and \( g(x) = \frac{1}{x-5} \).

Which of the following best describes the difference between the graphs of the two functions?

A. \( g(x) \) is a shift of \( f(x) \) to the left 5 units.
B. \( g(x) \) is a shift of \( f(x) \) to the right 5 units.
C. \( g(x) \) is a shift of \( f(x) \) up 5 units.
D. \( g(x) \) is a shift of \( f(x) \) down 5 units.

For Items 52 and 53, sketch a graph of each function.

52. \( h(x) = \frac{1}{5x + 3} \)

53. \( u(x) = -\frac{6}{x} - \frac{1}{2} \)

54. **Attend to precision.** Write to explain to another student how to obtain the graph of \( r(x) = \frac{1}{x + 1} + 1 \) from the graph of \( f(x) = \frac{1}{x} \).

55. **Express regularity in repeated reasoning.** Write to explain to another student the similarities in the graphs of \( m(x) = \frac{1}{x + a} + b \), \( n(x) = (x + a)^2 + b \), and \( o(x) = \sqrt{x + a} + b \) when compared to the graphs of their respective parent functions.
**LESSON 13-2**

56. What is the horizontal asymptote of \( f(x) = \frac{4x^2-1}{3x^2+2} \)?
   A. \( y = 0 \)
   B. \( y = \frac{1}{2} \)
   C. \( y = \frac{4}{3} \)
   D. \( y = \frac{3}{4} \)

57. What is the horizontal asymptote of \( g(x) = \frac{7x - 6x^4 + 3x^2}{9x^3 - x + 1 - 4x^4} \)?
   A. \( y = 0 \)
   B. \( y = \frac{7}{9} \)
   C. \( y = \frac{3}{2} \)
   D. \( y = \frac{1}{3} \)

58. **Make use of structure.** Find the equation of the slant asymptote of the function.
   \( h(x) = \frac{5x^2 - 3x + 1}{x - 1} \)

59. Find the vertical asymptote and the point discontinuity in the graph of the function.
   \( f(x) = \frac{x^2 - 2x + 3}{x^4 - 81} \)

60. **Attend to precision.** Sketch a graph of the function without using a graphing calculator.
   \( p(x) = \frac{3x + 4}{x^3 - 1} \)

**LESSON 13-3**

**Model with mathematics.** For Items 61–63, write a possible function whose graph could have the following asymptotes.

61. \( y = -4, x = 3 \)

62. \( y = 0, x = \pm 1 \)

63. \( y = -\frac{1}{2}x + 1, x = 0 \)
64. Which of the following functions could have a point of discontinuity at $x = 1$?

A. $f(x) = \frac{x}{x - 1}$
B. $f(x) = \frac{x - 1}{(x - 1)(x + 1)}$
C. $f(x) = \frac{x + 1}{(x + 1)(x - 1)}$
D. $f(x) = \frac{x^2}{x - 1}$

65. Critique the reasoning of others. Kayla says that a function that is a rational expression has only vertical asymptotes where the factors in the denominator equal zero. Is she correct? If not, explain and correct her error.